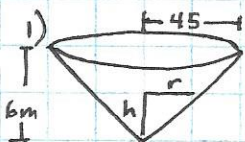


Related Rates 2



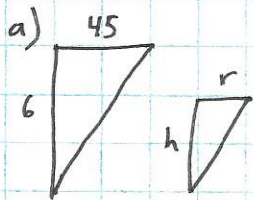
MOMENT

$$h = 5 \text{ m.}$$

RATES

$$\frac{dV}{dt} = -50 \text{ m}^3/\text{min}$$

$$V = \frac{1}{3} \pi r^2 h$$



$$\frac{r}{45} = \frac{h}{6}$$

$$r = \frac{15}{2} h$$

$$V = \frac{1}{3} \pi \left(\frac{15}{2} h \right)^2 h$$

$$V = \frac{225}{12} \pi h^3$$

$$\frac{dV}{dt} = \frac{225}{4} \pi (h^2) \frac{dh}{dt}$$

$$\left. \frac{dV}{dt} \right|_{h=5} = \frac{225}{4} \pi \cdot 25 \frac{dh}{dt}$$

$$-50 = \frac{225 \pi (25)}{4} \frac{dh}{dt}$$

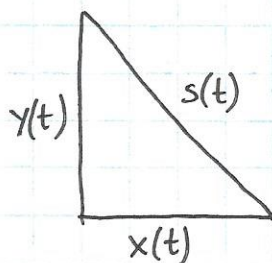
$$\frac{dh}{dt} = -\frac{8}{225 \pi} \text{ m/min}$$

$$\text{b) } \frac{dr}{dt} = \frac{15}{2} \cdot \frac{dh}{dt}$$

$$\frac{dr}{dt} = \frac{15}{2} \cdot \left(-\frac{8}{225 \pi} \right)$$

$$\frac{dr}{dt} = -\frac{4}{15 \pi} \text{ m/min}$$

2)



$$[x(t)]^2 + [y(t)]^2 = [s(t)]^2$$

MOMENT (3 seconds)

$$y'(t) = 65 + 1(3)$$

$$y(t) = 68$$

$$x(t) = 17(3) = 51$$

$$s(t) = 85$$

RATES

$$y'(t) = 1 \text{ ft/sec}$$

$$x'(t) = 17 \text{ ft/sec}$$

$$2(x(t)) \cdot x'(t) + 2(y(t))y'(t) = 2(s(t)) \cdot s'(t)$$

$$51(17) + 68(1) = 85 \cdot s'(t)$$

$$\underline{\underline{s'(t) = 11 \text{ ft/sec}}}$$

3) Cone: $V = \frac{1}{3}\pi r^2 h_1$
 Cylinder: $V = \pi r^2 h_2$

MOMENT
 $h_{\text{cone}} = 5 \text{ in}$

Rate
 $\frac{dV_{\text{cone}}}{dt} = -10 \text{ in}^3/\text{min}$

a) Cylinder ($r=3$ always)
 $V = 9\pi h_2$

$$\frac{dV}{dt} = 9\pi \frac{dh_2}{dt}$$

$$10 = 9\pi \frac{dh_2}{dt}$$

$$\underline{\underline{\frac{dh_2}{dt} = \frac{10}{9\pi} \text{ in/min}}}$$

b) Cone
 $V = \frac{1}{3}\pi r^2 h_1$
 $r = \frac{1}{2}h_1 \rightarrow V = \frac{1}{3}\pi \left(\frac{1}{2}h_1\right)^2 h_1$

$$V = \frac{1}{12}\pi h_1^3$$

$$\frac{dV}{dt} = \frac{1}{4}\pi h_1^2 \frac{dh_1}{dt}$$

$$-10 = \frac{1}{4}\pi (5)^2 \frac{dh_1}{dt}$$

$$-\frac{40}{25\pi} = \frac{dh_1}{dt}$$

$$\underline{\underline{\frac{dh_1}{dt} = -\frac{8}{5\pi} \text{ in/min}}}$$

4)

MOMENT

$x = 5$

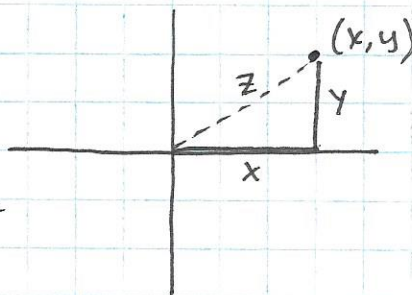
$y = 12$

$z = 13$

RATES

$\frac{dx}{dt} = -1 \text{ m/sec}$

$\frac{dy}{dt} = -5 \text{ m/sec}$



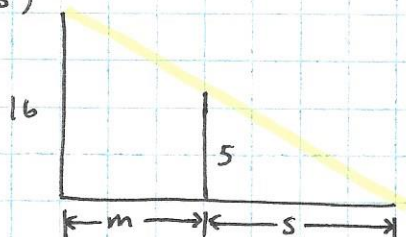
$x^2 + y^2 = z^2$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$5(-1) + 12(-5) = 13 \frac{dz}{dt}$$

$$\frac{dz}{dt} = -5 \text{ m/sec}$$

5)

MOMENT

?

RATE

$\frac{dm}{dt} = -5 \text{ ft/sec}$

a) $11 \frac{ds}{dt} = 5 \frac{dm}{dt}$

$$\frac{ds}{dt} = -\frac{25}{11} \text{ ft/sec}$$

$$\frac{s}{5} = \frac{s+m}{16}$$

$$16s = 5s + 5m$$

$$11s = 5m$$

b)
$$\underline{\underline{\frac{dm}{dt} + \frac{ds}{dt} = -5 + -\frac{25}{11} = -\frac{80}{11} \text{ ft/sec}}}$$